EE 576 - 3D Motion Estimation

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Time Varying Images

- Recovery of motion parameters from a sequence of images
- Relation between 3D motion of the objects and its projection on a 2D image frame.

Image Flow

Determined by the relative rigid-body and camera motions as well as the structure of the objects in the visibility range of the observer.

3D Motion and 2D Motion

Underlying Assumptions

- The motion parameters to be extracted are those of rigid body motion
- Uniform incident illumination which enables the assumption that brightness is a function of the reflectance of the object
- 3D motion corresponds directly to changes in the image brightness and therefore alone by processing the image intensities, the motion parameters can be extracted.

Image Brightness Change Equation

$$\begin{bmatrix} D_{x_1} I \\ D_{x_2} I \end{bmatrix}^T \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = -D_t I$$

Letting

$$D_x I = \begin{bmatrix} D_{x_1} I \\ D_{x_2} I \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

then,

$$D_x I^T \dot{x} = -D_t I$$

Frame Transformations

Let a point P_j be specified in a frame F_1 with the coordinates:

$$^1P_j^T = \left[\begin{array}{ccc} ^1\pi_1 & ^1\pi_2 & ^1\pi_3 \end{array} \right]$$

Namely,

$$^1P_j =^1 R_j P_j +^1 r_j$$

Linear Velocity

In order to compute the linear velocity of the point ${}^1P_j$, we differentiate this equation wrt to time. With the rigid body assumption,

$$ {}^1\dot{P}_j = {}^1\dot{R}_j P_j + {}^1\dot{r}_j $$

Using the fact that $\dot{R}_j = J(\omega)R_j$, the equation becomes

$$ {}^1\dot{P}_j = J(\omega){}^1R_j^1P_j + {}^1\dot{r}_j $$

where ω are the angular velocity components. Noting that ${}^1R_j^1P_j = {}^1P_j - {}^1r_j$,

$$ {}^1\dot{P}_j = J(\omega)^1\left({}^1P_j - {}^1r_j\right) + {}^1\dot{r}_j \quad (1) $$

Linear Velocity

- Two components introduced by angular velocity to linear velocity.
- Letting ${}^1t_j = -J(\omega){}^1r_j + {}^1\dot{r}_j$, and using the isomorphism between vectors and skew-symmetric matrices,

$$
{}^1\dot{P}_j = (\omega \times {}^1P_j) + {}^1t_j
$$

2D Transformation

The corresponding image plane coordinates of a P_j specified in a frame F_1 is:

$$p_{j,1} = \begin{bmatrix} \mu_{j,1} \\ \nu_{j,1} \end{bmatrix}$$

where

$$\begin{aligned} \mu_{j,1} &= \frac{{}^1\pi_1}{{}^1\pi_3} \\ \nu_{j,1} &= \frac{{}^1\pi_2}{{}^1\pi_3} \end{aligned}$$

Image Flow Equation

The image flow equation is derived by differentiation wrt to time as:

$$\dot{p}_{j,1} = \frac{d}{dt}\begin{bmatrix} \frac{^1\pi_1}{^1\pi_3} \\ \frac{^1\pi_2}{^1\pi_3} \end{bmatrix}$$

Namely,

$$\dot{p}_{j,1} = \begin{bmatrix} \frac{^1\dot{\pi}_1 {^1\pi_3} - {^1\pi_1} {^1\dot{\pi}_3}}{(^1\pi_3)^2} \\ \frac{^1\dot{\pi}_2 {^1\pi_3} - {^1\pi_2} {^1\dot{\pi}_3}}{(^1\pi_3)^2} \end{bmatrix}$$

2D Transformation

Noting that

$$ {}^1P_j \times \dot{P}_j = \begin{bmatrix} {}^1\pi_2 {}^1\dot{\pi}_3 - {}^1\dot{\pi}_2 {}^1\pi_3 \\ -{}^1\pi_1 {}^1\dot{\pi}_3 + {}^1\dot{\pi}_1 {}^1\pi_3 \\ {}^1\pi_1 {}^1\dot{\pi}_2 - {}^1\dot{\pi}_1 {}^1\pi_2 \end{bmatrix} $$

we can write

$$ \dot{p}_{j,1} = \frac{1}{({}^1\pi_3)^2} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} J({}^1P_j)^1\dot{P}_j $$

$$ \dot{p}_{j,1} = \frac{1}{({}^1\pi_3)^2} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} J({}^1P_j) \left(\left(\omega \times {}^1P_j \right) + {}^1t_j \right) $$

Replacing the terms appearing in $\frac{1}{{}^1\pi_3} J({}^1P_j)$ by the image coordinates

$$\dot{p}_{j,1} = \frac{1}{{}^1\pi_3} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} J(\begin{bmatrix} \mu_{j,1} \\ \nu_{j,1} \\ 1 \end{bmatrix}) \left((\omega \times {}^1 P_j) + {}^1 t_j \right)$$

Also noting that

$$\frac{1}{{}^1\pi_3} \left((\omega \times {}^1 P_j) + {}^1 t_j \right) = -J\left(\begin{bmatrix} \mu_{j,1} \\ \nu_{j,1} \\ 1 \end{bmatrix} \right) \omega + \frac{1}{{}^1\pi_3} {}^1 t_j$$

$$\dot{p}_{j,1} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} J(\begin{bmatrix} \mu_{j,1} \\ \nu_{j,1} \\ 1 \end{bmatrix}) \left(-J\left(\begin{bmatrix} \mu_{j,1} \\ \nu_{j,1} \\ 1 \end{bmatrix} \right) \omega + \frac{1}{{}^1\pi_3} {}^1 t_j \right)$$

Stereo and Time-Varying Images

- If a scene is observed from two cameras over a time period, how the two time-varying image frame sequences could be used together to derive the motion parameters?
- As a textured rigid object moves through space, the evolving image sequences registered by two cameras contain information in the form of image flow field.