EE 576 - Light & Image Formation

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Light & Image Formation
Reflectance Models
Shading

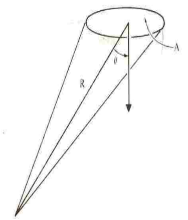
Image Formation: What we see?

The image of a three-dimensional object depends on:

1. Light sources
2. Object - Shape & material
3. Sensor - Eye, camera

Solid angle subtended by a patch

- Proportional to its area A and the cos of the angle of inclination θ
- Inversely proportional to the distance R as $\Omega = \frac{A cos\theta}{R^2}$.

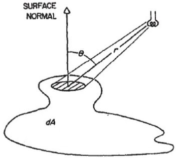


Apparent area: Surface area multiplied by the cosine of the angle between the surface normal and the specified direction.

Radiance & Irradiance

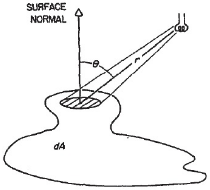
- **Radiance**: The amount of light emitted from a surface as measured by power per unit unit area per unit solid angle. ($Wm^{-2}\ sr^{-1}$ – watts per square meter per steradian)
- Brightness - Amount of energy per unit apparent area
- **Irradiance**: The amount of light falling on a surface as measured by power per unit area (Wm^{-2} – Watts per square meter)

Radiance & Irradiance (Source $\rightarrow$ Object)



Flux	Φ	W
Radiant Intensity	$I = \frac{d\Phi}{d\omega}$	W sr^{-1}
Irradiance	$E = \frac{d\Phi}{dA}$	$W\ m^{-2}$
Radiant existance	$M = \frac{d\Phi}{dA}$	$W\ m^{-2}$
Radiance	$L = \frac{d^2\Phi}{dAd\Phi cos\theta}$	$W\ m^{-2}$ sr $^{-1}$

Radiance & Irradiance (Source → Object)



Solid angle $d\omega = \frac{dA\cos\theta}{r^2}$

Flux intercepted by dA $d\Phi = Id\omega = I\frac{dA\cos\theta}{r^2}$

Irradiance of dA $E = \frac{d\Phi}{dA} = \frac{I\cos\theta}{r^2}$

Radiance & Irradiance (Object to Camera)

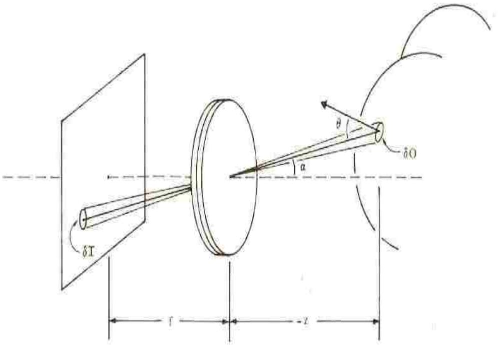


Image Formation

Rays passing through the center of the lens are not deflected.

$\rightarrow$ Solid angle of object $=$ Solid angle of corresponding image patch.

Solid angle of object patch δO as seen from lens center - $\frac{\delta O cos\theta}{(z/cos\alpha)^2}$

Solid angle subtended by image patch δI - $\frac{\delta I cos\alpha}{(f/cos\alpha)^2}$

Equality $\Rightarrow \frac{\delta O}{\delta I} = (\frac{z}{f})^2 \frac{cos\alpha}{cos\theta}$

Object Radiance

Amount of light emitted by surface that makes its way through lens
(Solid angle of lens as seen from surface patch)

$$\Omega = \frac{(\frac{d}{2})^2 \pi cos\alpha}{(z/cos\alpha)^2}$$

Power of light originating from the object patch and passing through the lens is

$$\delta P = L \delta O cos\theta \Omega = L \delta O \frac{(\frac{d}{2})^2 \pi cos\alpha cos\theta}{(z/cos\alpha)^2}$$

Image Irradiance

The resulting image irradiance of the image patch:

$$E = \frac{\delta P}{\delta I} = L\frac{\delta O}{\delta I}\Omega cos\theta = L\delta O\frac{(\frac{d}{2})^2\pi cos\alpha}{(z/cos\alpha)^2} = L\frac{\pi}{4}(\frac{d}{f})^2 cos^4\alpha$$

Next Question – Scene Radiance

- The amount of light falling on a surface E
- The fraction of light that is reflected as well as viewing angle L

Object - Reflectance Model

Reflectance model - Mathematical formulation of how light energy incident on an object is transferred from the object to the camera sensor.

Bidirectional Reflectance Distribution Function – BRDF

A measure on the observed brightness from a certain direction θ_e when the light is incident in a different direction θ_i :

$$f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{\delta L(\theta_e, \phi_e)}{\delta E(\theta_i, \phi_i)}$$

Bidirectional reflectance distribution function

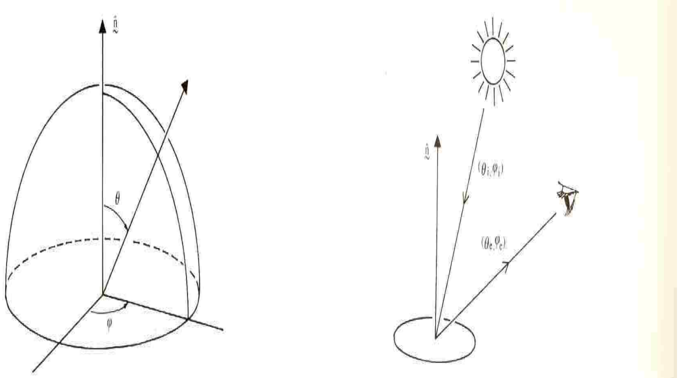
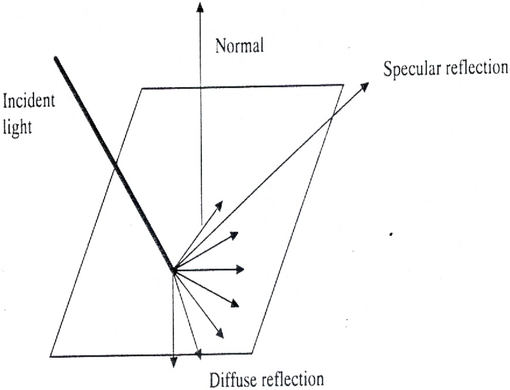


Figure: Left: Spherical coordinate system; Right: Bidirectional reflectance distribution function.

BRDF Models

- Lambertian – Equally bright from all directions
- Specular – The surface luminance is highest when the observer is located at the perfect reflection direction, and falls off sharply.
- Phong – A mixture of Lambertian and Specular

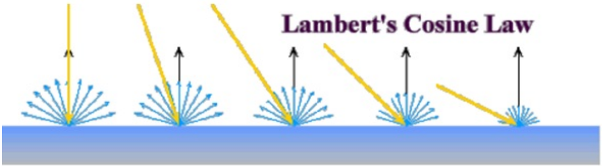
Light Reflection from a Surface



Lambertian Surfaces

Apparent brightness of the surface to an observer - The same regardless of the observer's angle of view

- $L = E\rho cos\theta$
- ρ is the albedo



Specular Reflection

The surface luminance is highest when the observer is situated at the perfect reflection direction, and falls off sharply.

- Law of reflection $\rightarrow$ The angle of incidence equals the angle of reflection.
- The incident, normal, and reflected directions – coplanar.

Phong Model

Lambertian reflection is typically accompanied by specular reflection.

- $L = E(acos\theta + bcos^n\alpha)$
- a – Diffuse albedo
- b – Specular albedo

Phong Reflection

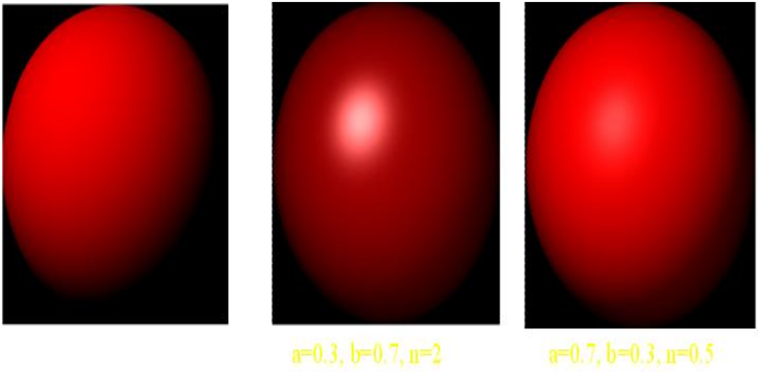
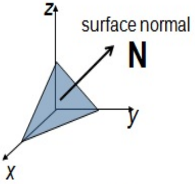


Figure: Shading under different models - Left: Lambertian only, Center: $a = 0.3, b = 0.7, n = 2$, Left: $a = 0.7, b = 0.3, n = 0.5$

Surface Normals



Equation of plane $Ax + By + Cz + D = 0$

or $\frac{A}{C}x + \frac{B}{C}y + z + \frac{D}{C} = 0$

Let $-\frac{\partial z}{\partial x} = \frac{A}{C} = p \qquad -\frac{\partial z}{\partial y} = \frac{B}{C} = q$

Surface normal

$$\mathbf{N} = \left(\frac{A}{C}, \frac{B}{C}, 1\right) = (p, q, 1)$$

Surface Normals (cont)

Surface normal at each point on the surface – Expressed as the directional slope in the X_1 and X_2 directions.

$$\begin{bmatrix} f_{X_1} \\ f_{X_2} \\ -1 \end{bmatrix} = \begin{bmatrix} p \\ q \\ 1 \end{bmatrix}$$

Normalized Surface Normals

p, q – Gradient or gradient space representation for local surface orientation

Normalized surface normal

$$\frac{1}{\sqrt{1+p^2+q^2}}\begin{bmatrix} p \\ q \\ 1 \end{bmatrix}$$

Gradient Space

- The $p - q$ plane is called the gradient space and every point corresponds to a particular surface orientation.
- The point at the origin represents the orientation of all planes normal to the viewing direction
- A contour map – Depict the reflectance map.

References

[1]

R. A. Horn and R. W. Sjoberg, "Calculating the Reflectance Map," *Applied Optics*, vol. 18, no. 11, pp. 1770–1779, 1979.