EE 576 - Optical Flow

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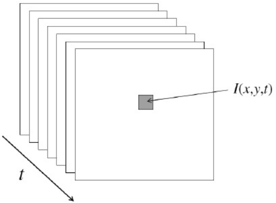
May 20, 2020

Motion Analysis
- Introduction
- Optical Flow

Time-varying Images

- Images: Two dimensions – namely the image plane coordinates.
- Time varying images – time as 3^{rd} dimension
- Single image $\rightarrow$ A sequence of images.
- Imaging devices $\rightarrow$ Images sampled over time $\rightarrow$ Discrete nature.



Processing Time-Varying Images

New possibilities

- Extracting motion information
- Object detection
  - Camouflaged objects - Only easily seen when they move.
  - Relative sizes and position of objects - More easily determined when the objects move.
  - Even simple image differencing – Edge detector for the silhouettes of texture-free objects moving over any static background.

Difficulties

- Additional storage and processing requirements

Motion Types

Different types of motion to consider:

- Motion of objects only (static camera)
- Motion of camera (ego motion) only
- Both objects and camera moving

Effects of motion

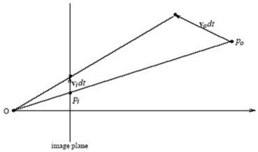
- Since motion is relative $\rightarrow$ Expect the results of these different types of motion to be same.
- Not true!
- Movement relative to the illumination source, specularity or shadow effects may be changing!

Analysis of Visual Motion

- Measurement of image motion (dense optical flow)
- Tracking (sparse optical flow)
- Extract 3D information about the shape & motion of the objects
- Detection of discontinuities - Segmenting the scene into distinct objects

Moving Objects

- Consider an object point p_o moving with velocity v_o
- Corresponding image point p_i moves on the image plane with velocity vector v_i
- In case of rigid body translations and rotations $\rightarrow$ Cont. motion field except at object boundaries



Optical Flow

Optical flow: Apparent motion of brightness patterns observed when a camera is moving relative to the objects being observed.

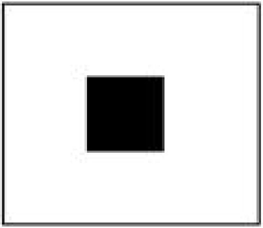


Image 1

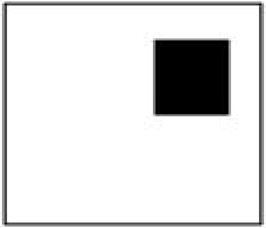
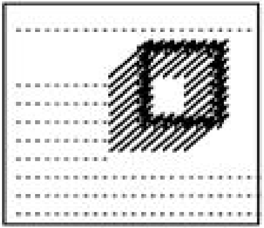


Image 2



Part of motion field

Pure Camera Translation

- *Focus of expansion/contraction:*(FOE/FOC) The image point from which (or towards which) all motion vectors radiate
- Point of divergence (or convergence) of all motion field vectors.
  - Forward motion of camera → Divergence
  - Backwards motion → Convergence
- Direction of motion - Along the projection ray through FOE.

Focus of Expansion

Time-invariant Illumination

Intensity function $I : X \times R \rightarrow C$ as $I(x_1, x_2, t) \in C$

$$I(x, y, t + \delta t) \;=\; I(x, y, t) + \frac{dI}{dt} + HOT \;\cong I(x, y, t) + \frac{dI}{dt}$$

Time-invariant illumination $\rightarrow I(x, y, t + \delta t) = I(x, y, t)$

$$\frac{dI}{dt} \;\cong\; 0$$

Now consider $\frac{dI}{dt}$

$$\frac{dI}{dt} = \frac{\partial I}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial I}{\partial x_2}\frac{dx_2}{dt} + \frac{\partial I}{\partial t} = 0$$

Optical Flow & Motion Field

Optical flow ≈ Motion field

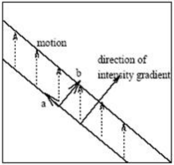
Exceptions!

Aperture problem 7

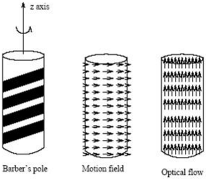
Measure optical flow

along intensity gradient!

In general, assume identicalness



Barber's Pole



https://en.wikipedia.org/wiki/Barberpole_illusion

Optical Flow Equation

Let

$$\dot{x}_1 = \frac{dx_1}{dt} \qquad \dot{x}_2 = \frac{dx_2}{dt}$$

$$D_{x_1} I \dot{x}_1 + D_{x_2} I \dot{x}_2 + D_t I = 0$$

$$\boxed{\begin{bmatrix} D_{x_1} I \\ D_{x_2} I \end{bmatrix}^T \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = -D_t I}$$

How many equations and unknowns per pixel?

- One equation and 2 unknowns

Optical Flow Estimation

- *Underconstrained* $\Longrightarrow$ No unique solution

- Enforcing some smoothness constraint over neighbouring pixels
  - Regularization - Euler-Lagrange equations
  - Fixed neighborhood - ie. 3×3 neighborhood
    - Small motion - Pixels do not move very far
    - Spatial coherence - Pixels move like neighbors

Regularization

Consider $u : R^2 \to R$ and $v : R^2 \to R$ as functions and minimize

$$\int \int F(u, v, u_{x_1}, u_{x_2}, v_{x_1}, v_{x_2}, \lambda)$$

$$F(u, v, u_{x_1}, u_{x_2}, v_{x_1}, v_{x_2}, \lambda) = u_{x_1}^2 + u_{x_2}^2 + v_{x_1}^2 + v_{x_2}^2 + \lambda(D_{x_1} Iu + D_{x_2} Iv + D_t I)^2$$

Corresponding Euler equations are:

$$\begin{aligned} D_u F - \frac{\partial}{\partial x_1} D_{u_{x_1}} F - \frac{\partial}{\partial x_2} D_{u_{x_2}} F &= 0 \\ D_v F - \frac{\partial}{\partial x_1} D_{v_{x_1}} F - \frac{\partial}{\partial x_2} D_{v_{x_2}} F &= 0 \end{aligned}$$

Continuous Differential Equations

$$\begin{aligned}\nabla^2 u &= -\lambda(D_{x_1} I u + D_{x_2} I v + D_t I) D_{x_1} I \\ \nabla^2 v &= -\lambda(D_{x_1} I u + D_{x_2} I v + D_t I) D_{x_2} I\end{aligned}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$$

Iterative Solution

$$u_{ij}^{n+1} = \overline{u}_{ij}^{n} + \frac{D_{x_1} I \overline{u}_{ij}^{n} + D_{x_2} I \overline{v}_{ij}^{n} + D_t I}{1 + \lambda(D_{x_1} I^2 + D_{x_1} I^2)} D_{x_1} I$$

$$v_{ij}^{n+1} = \overline{v}_{ij}^{n} + \frac{D_{x_1} I \overline{u}_{ij}^{n} + D_{x_2} I \overline{v}_{ij}^{n} + D_t I}{1 + \lambda(D_{x_1} I^2 + D_{x_1} I^2)} D_{x_2} I$$

Average $\overline{u}_{ij}^{n}$ - Computed using stencil:

$$\frac{1}{20} \begin{matrix} 1 & 4 & 1 \\ 4 & 0 & 4 \\ 1 & 4 & 1 \end{matrix}$$

Iterations until convergence!

Summary

- New value of (u, v) - Equal to the average of the surrounding values minus an adjustment in the direction of the brightness gradient – using estimates of the values $D_{x_1} I$, $D_{x_2} I$ and $D_t I$.
- Taking local averages in neighbourhoods about the grid point (i; j; k).

Fixed Neighborhoods

Lucas-Kanade Flow: $N \times N$ neighborhoods $\rightarrow i = 1, \ldots, N^2$

$$\begin{bmatrix} D_{x_1} I(p_1) & D_{x_2} I(p_1) \\ \vdots & \vdots \\ D_{x_1} I(p_9) & D_{x_2} I(p_9) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_1 \end{bmatrix} = - \begin{bmatrix} D_t I(p_1) \\ \vdots \\ D_t I(p_9) \end{bmatrix} \rightarrow Au = b$$

Least squares solution

$$\begin{bmatrix} \sum_i D_{x_1} I(p_i) D_{x_1} I(p_i) & \sum_i D_{x_1} I(p_i) D_{x_2} I(p_i) \\ \sum_i D_{x_1} I(p_i) D_{x_2} I(p_i) & \sum_i D_{x_2} I(p_i) D_{x_2} I(p_i) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_1 \end{bmatrix} = - \begin{bmatrix} \sum_i D_{x_1} I(p_i) D_t I(p_i) \\ \sum_i D_{x_2} I(p_i) D_t I(p_i) \end{bmatrix}$$

$(A^T A)$ invertible (eigenvalues $\lambda_1 \geq \lambda_2$ ¿ 0 !) and well-conditioned $(\frac{\lambda_1}{\lambda_2} < \infty)$

$\rightarrow u = (A^T A)^{-1} A^T b$