EE 576 - Enhancement

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Why?

Incoming images subjected to various types of processing
All aimed at improving the interpretability or perception of information in images

- Spatial domain methods that operate directly on the spatial domain
- Frequency domain methods that operate on the frequency domain

What constitutes a good image with either machine or human perception?
If it provides robust information or if it looks good

Spatial Domain Methods

- To obtain an enhanced image $\check{I}$
- An improved version of the original image I based on maps T of intensity of the pixel or the intensities of pixels in its neighborhood.
- Neighbourhoods – of any shape, but usually a rectangular form

Gray Scale Manipulation

Simplest: Binary images: The intensity profile subjected to a step function with a certain threshold τ:

$$T(x) = \begin{cases} 0 & \text{if} \quad I(x) < \tau \\ 1 & \text{otherwise} \end{cases}$$

Mathematical Definition

- Binary (two-valued) images.
- Intensity values – Quantized to two values, usually denoted 0 and 1,
- Pixel values 0 and 255, representing black and white.
- Binary transformation map $b : C \rightarrow \{0, 1\}$
- Thresholded image $b \circ I : X \rightarrow \{0, 1\}$
- For each $x \in X$,

$$b(x) = \begin{cases} 1 & \text{if belongs to an object(s) of interest} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Sample Binary Image

Figure: A thresholded image

Advantages

- Simplest to process.
- Outline of the object which is itself also easily obtainable.
- Some sample application domains
  1. Identifying objects, for example, sorting packages
  2. Identifying orientations
  3. The output of other image processing techniques
  4. Text processing

Threshold Operator

- Typically obtained by thresholding a gray level image.
- Threshold operator

$$b(x) = \begin{cases} 1 & \text{if } I(x) > \tau \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

- A white object on a black background (or vice versa, depending on the relative gray values of the object and the background).
- The 'negative' of a binary image – a binary image However, choosing a threshold is not obvious.
- Histogram of the number of times each gray level occurs in the image.
- Bimodal histogram $\rightarrow$ Selection of τ is easy

Thresholded Image

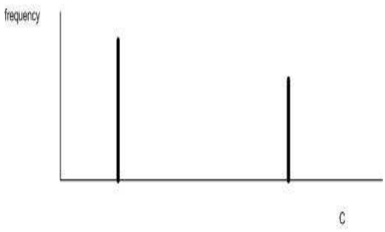


Figure: Ideal histogram for thresholding

Binary Image

Figure: Applying threshold operator

Binary Image

Figure: Applying threshold operator

Histograms & Thresholding

- Histogram – not bimodal.
- Measurement noise $\Rightarrow$ Convolving the 'ideal' histogram with the probability distribution of the noise.
- Gray levels of the object and the background – Fairly close to each other, the case of noise $\rightarrow$ A shoulder in the histogram.
- $\rightarrow$ Histogram – not bimodal!!
- No clear way of choosing the threshold.

Binary Image

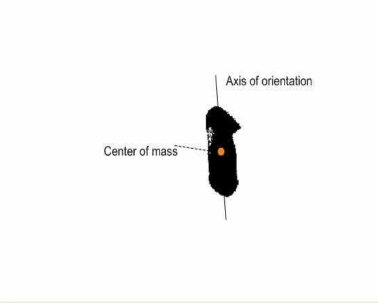
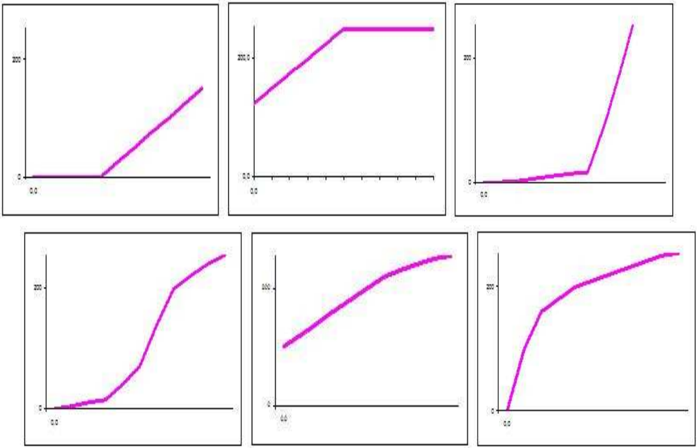


Figure: Applying threshold operator

Simplest form of operation - The map T acts only on the pixel intensity $I(x)$ – namely gray scale maps.



Aim

Minimize camera noise, compenstate for spurious or missing pixel values.
Use convolution with a proper filter f

$$I(x) * f(x) = \int\int I(x_1 - \tau_1, x_2 - \tau_2) f(\tau_1, \tau_2) d\tau_1 d\tau_2 \qquad (3)$$

In discretized version

$$I(x) * f(x) = \sum_{\tau_1} \sum_{\tau_2} I(x_1 - \tau_1, x_2 - \tau_2) f(\tau_1, \tau_2) \qquad (4)$$

Neighbourhood Averaging

Use the weighted average intensity value in a neighborhood

$$T(x) = A_x * I(x) = \sum_{y \in N_\epsilon(x)} A_x(y) I(x-y)$$

A is the 3×3 matrix centered at point x. In straightforward averaging, A is taken as follows:

$$A_x = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Averaging

- Due to spatial coherence, the value of a pixel – more closely related to the values of pixels close to it than to those further away.
- Not valid near edges.
- Instead of uniform weighting, use a weighting scheme that favors the pixels near the centre of the mask more strongly than those at the edge.
- Some common weighting functions - Triangular, Gaussian.
- In common practice, Gaussian smoothing – most frequently
- In practice, not much difference
- Smoothing attenuates high frequency components,
- ask shapes other than Gaussian – odd effects on the spectrum

Median Filering

Neighbourhood averaging – Blur edges
Remedy: Median filtering –
    The median of the pixel values in the neighbourhood of that pixel.
The median: Half the values in the set are less than m and half are greater.
The outcome of median filtering – Outliers become more like their neighbours while edges are preserved.
Median filters - non-linear.

Sharpening

Aims to highlight fine detail in the image, or to enhance detail that has been blurred possibly due to noise, motion or other factors.

Sharpening – just the opposite of smoothing

High frequency components accentuated $\rightarrow$ Spatial filter shape with a high positive component at the centre.

High pass filtering - Subtracting a low pass image from the original image

Sharpening

$$Highpass = Original - Lowpass$$

$$\begin{bmatrix} -1/9 & -1/9 & -1/9 \\ -1/9 & 8/9 & -1/9 \\ -1/9 & -1/9 & -1/9 \end{bmatrix}$$

All the weights add up zero, the resulting signal will have a zero DC value

Equivalently the coefficient of the zero frequency term in the Fourier expansion is zero.

High boost Filtering

High pass image required $\rightarrow$ The low frequency components should not be lost.
High boost filtering – Original image is multiplied by an amplification factor a before subtracting the low pass image:

High boost Filtering

$$
\begin{aligned}
Highpass &= aOriginal - Lowpass \\
&= (a-1)Original + Original - Lowpass \\
&= (a-1)Original + Highpass
\end{aligned}
$$

If $a = 1$, the result is simply the high pass filtered image. However, if $a > 1$, then part of the original image is preserved.

$$
\begin{bmatrix}
-1/9 & -1/9 & -1/9 \\
-1/9 & w/9 & -1/9 \\
-1/9 & -1/9 & -1/9
\end{bmatrix}
$$

where $w = 9a - 1$.

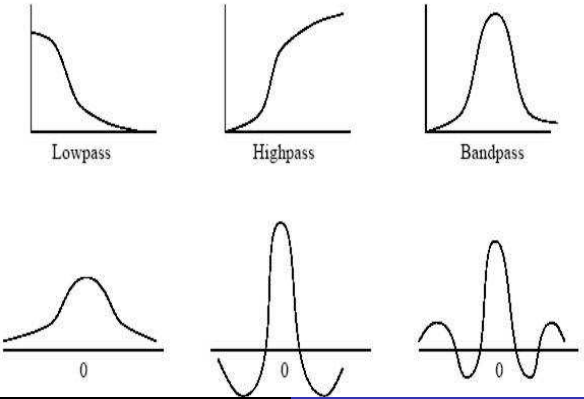
Fourier Analysis

- Rather than convolve in the spatial domain
- Frequency domain – Fourier transform of the image to be enhanced is computed which is then multiplied by the frequency transformed filter and followed by inverse transform operation.
- Blurring an image means reducing its high frequency components, or sharpening an image means increasing the magnitude of its high frequency components.
- Computationally, more efficient to implement these operations as convolutions in the spatial domain.
- Frequency domain concepts

Filtering

- Low pass filtering - The elimination of the high frequency components in the image.
- Blurring of the image (and thus a reduction in sharp transitions associated with noise).
- An ideal low pass filter – Retain all the low frequency components, and eliminate all the high frequency components.
- However, ideal filters suffer from two problems: blurring and ringing.
- Due to the shape of the associated spatial domain filter, which has a large number of undulations.
- Smoother transitions in the frequency domain filter, such as the Butterworth filter – much better results.

Spatial Filters



Homomorphic Filter

The basic nature of the image I may be characterized by two components:

1. Illumination – The amount of source light incident on the scene being viewed (L)
2. Reflectange – The amount of light reflected by the objects in the scene. (R)

Hence,

$$I(x) = L(x)R(x)$$

where $0 < L(x) < \infty$ and $0 < R(x) < 1$. If we apply *ln* operator,

$$ln(I(x)) = ln(L(x)) + ln(R(x))$$

which means that

Homomorphic Filter

Equivalently,

$$I(\omega) = L(\omega) + R(\omega)$$

First component – Low frequency illumination term
Second component – the high frequency reflectance term.

Homomorphic Filter (cont.)

A filter that suppresses low frequency components $\rightarrow$ Suppress the illumination component and enhance the reflectance component:

$$T(\omega) = H(\omega)L(\omega) + H(\omega)R(\omega)$$

In the spatial domain,

$$T(x) = F^{-1}(H(\omega)L(\omega)) + F^{-1}(H(\omega)R(\omega))$$

Taking the exponential of the resulting image,

$$\breve{I}(x) = exp(F^{-1}(H(\omega)L(\omega)))exp(F^{-1}(H(\omega)R(\omega))$$

Homomorphic Filter

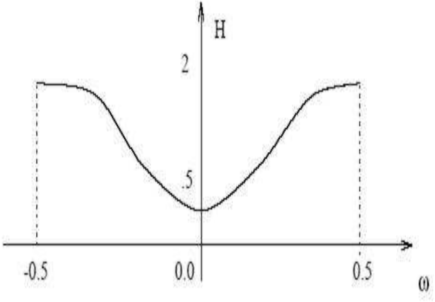


Figure: Homomorphic filter.

Geometric Transformations

Geometric transformations $t : P \to P$ such as:

- Rotation
- Scaling
- Distortion

Applying Geometric Transformations

Two steps:

1. Spatial transformations for spatially rearranging the pixels
2. Gray level interpolation to reassign intensity values

Spatial Transformation

Rotation about the origin by an angle θ

$$x' = Rx$$

$$R = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \quad (5)$$

Interpolation

- Geometric transformations $\rightarrow$ pixel values that are not integers.
- The gray level values are not at integer positions in the image
- True assignment of pixel values need to be determined.

Interpolation (cont.)

The simplest approach: Assign the gray value for x to the pixel having closest integer coordinates to x'.

Problem: Some pixels assigned multiple gray values while not assigned a gray level at all!

An alternative approach: Inverse image transformation.

Consider integer pixel locations in the output image and calculate where they must have come from in the input image.

Gray levels of the 4 surrounding integer pixel positions.

Interpolate across these known intensities to determine the correct gray level of the position when the output pixel came from.

$$I(x) = c_1 x_1 + c_2 x_2 + c_3 x_1 x_2 + c_4 \quad (6)$$

Mathematical Morphology

A set-theoretic method of image analysis.
Capable of providing a quantitative description of geometrical structures $\rightarrow$ Useful in extracting image components related to representation and description.

- Determination of boundaries of objects, their skeletons, and their convex hulls.
- Edge thinning and pruning.
- Primarily on binary and sometimes on gray level images

Set operations

- Morphological operations – Operations that enable simple expansion and shrinking.
- Consider $A, B \subset X$.
- A is a region of interest
- B is a structuring set – referred to as the structuring element.
- Typically the structuring element B is a circular disc in the plane- however this is not required.
- Furthermore, both sets could be defined in higher dimensions.

Set operations

- The **translation of A by x** is denoted A_x and is defined as

$$A_x = \{c \,|c = a + x, \forall a \in A\} \tag{7}$$

- The **reflection of** B is $\widehat{B}$

$$\widehat{B} = \{x \,|x = -b, \forall b \in B\} \tag{8}$$

- The **complement of A** is denoted A^C, and the difference of two sets A and B is denoted $A - B$.

Dilation & Erosion

1. Dilation:

$$A \oplus B = \left\{ x \,\middle|\, \widehat{B_x} \cap A \neq \emptyset \right\} \tag{9}$$

The result - A new set made up of all points generated by obtaining the reflection of B about its origin and then shifting this reflection by x.
If A is a rectangular image region and B is a disc centered on the origin. In this case, $\widehat{B_x} = B$ (since B is symmetric).

2. Erosion:

$$A \ominus B = \left\{ x \,\middle|\, \widehat{B_x} \subseteq A \right\} \tag{10}$$

Dilation & Erosion

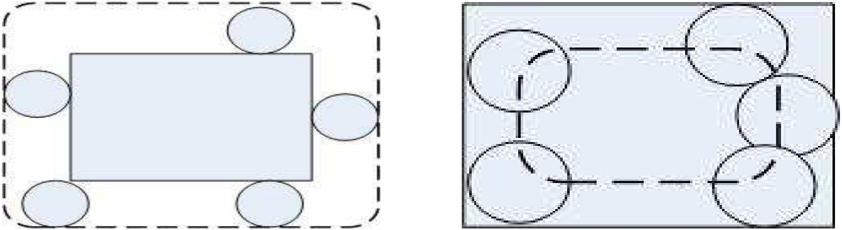


Figure: Result of morphological operations as shown by dashed regions – Left: Dilation; Right: Erosion

Opening and Closing

Two very important transformations are opening and closing.

- Opening: $A \circ B = (A \ominus B) \oplus B$
- Closing: $A \bullet B = (A \oplus B) \ominus B$

Opening

- Like 'rounding from the inside' – it can be shown that

$$A \circ B = \cup \left\{ B_x \middle| \widehat{B_x} \subseteq A \right\} \tag{11}$$

- The opening of A by B – Taking the union of all translates of B that are inside A while parts of A that are smaller than B are removed.
- Opening generally smooths a contour in an image as well as eliminating thin protrusions.

Opening (cont.)

It satisfies the following:

1. $A \circ B \subset A$
2. $C \subset A \Longrightarrow C \circ B \subset A \circ B$
3. $(A \circ B) \circ B = A \circ B$

Closing

- Like ‘smoothing from the outside’.
- Holes are filled in and narrow valleys are ‘closed’.
- Narrows smooth sections of contours, merging narrow breaks and long thin gulfs, eliminating small holes, and filling gaps in contours.

Consider sample image region with a circular structuring element.

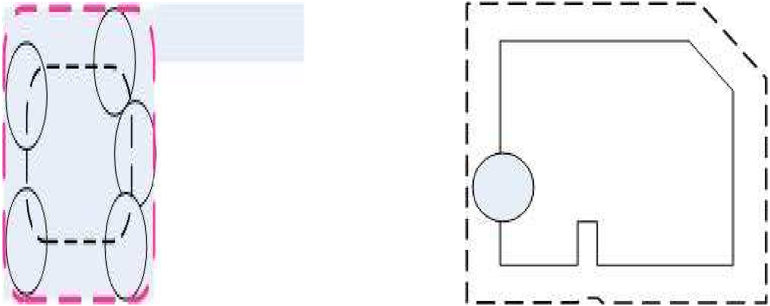


Figure: Result of morphological operations as shown by dashed regions – Left: Opening; Right: Closing

The following hold:

1. $A \subset A \bullet B$
2. $C \subset A \Longrightarrow C \bullet B \subset A \bullet B$
3. $(A \bullet B) \bullet B = A \bullet B$

- The third property - *idempotency*.
- Idempotency $\rightarrow$ Any application of the operation more than once will have no further effect on the result.
- It should be noted that opening and closing are duals of each other with respect to set complementation:

$$(A \bullet B)^C = A^C \circ B^C \tag{12}$$

General Comments

- Morphological operations preserve the main geometric structures of the object,
- Only features ‘smaller than’ the structuring element are changed by transformations,
- All other features at ‘larger scales’ are not changed significantly.

Applications

- Noise removal
- Boundary
- Region Filling
- Region Finding – Connected component
- Skeletonization

Noise Removal

The morphological filter $(A \circ B) \bullet B$ – To eliminate random, uniformly distributed small noisy elements – commonly known as 'salt and pepper' noise (appears as black dots or small blobs on a white background or vice versa)

- Assuming that all noise components are physically smaller than the structuring element B,
- The background noise is eliminated at the erosion stage,
- Erosion on its own will increase the size of the noise components on the object,
- These are eliminated at the closing operation.

Obtaining Boundary

The boundary $\delta A$of a set A, can be obtained by

- First eroding A with B, where B is a suitable structuring element,
- Performing the set difference between A and its erosion.

That is

$$\delta A = A - (A \ominus B)$$

A typical B

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (13)$$

Region Filling

Accomplished iteratively using dilations, complementation, and intersections.

$X_0 = \{x\}$
$X_k = (X_{k-1} \oplus B) \cap A^C$
If $X_k = X_{k-1}$ stop, else go to step 2

0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	0	0	1	0
0	0	1	1	1	0
0	0	0	0	0	0

0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	0	0	1	0
0	0	1	1	1	0
0	0	0	0	0	0

Figure: Filling of a region - Left: Original; Right: After filling.

Median Filtering

- Median filtering - a morphological operation.
- Erosion corresponds to having pixel values replaced with the smallest value in the neighbourhood while dilation is the exact opposite – that is pixel values are replaced with the largest value in their neighborhood.
- Median filtering replaces pixels with the median value in the neighbourhood. It is the rank of the value of the pixel used in the neighbourhood that determines the type of morphological operation.