EE 576 - Object - Single Image Viewing

H.I. Bozma

Electric Electronic Engineering
Bogazici University

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Single Image Viewing

Introduction
Vanishing Points and Lines
Projective Invariants
Rectification
Introduction

Human beings – Look at a photograph or a well executed painting and deduce considerable 3D information.

Figure: Single image viewing.
Perspective Projection

- First developed during the Italian Renaissance around 1430.
- A sense of three dimensionality in paintings.
- Establishing vanishing points and vanishing lines – both fundamental operations in reconstructing 3D information from a perspective image as follows:
Vanishing Points

- A set of parallel lines meet at a point.
- Two sets of parallel lines in different directions will give two vanishing points.
- These two vanishing points form a vanishing line for the collection of parallel planes defined by these two sets of parallel lines.
- Geometrically, the vanishing point is the intersection point of the image plane with one of these parallel lines that passes through the optical center of the camera.
An example of Vanishing Point

A number of parallel lines and their vanishing point $V$ on the image plane. These parallel lines in 3D – not on the same plane. The line $OV$ that is in the same direction as lines $AB$, $CD$, and $EF$ and (contains the optical centre $O$) intersects the image plane at the vanishing point $V$. 

![Diagram showing vanishing point and parallel lines](image.png)
Locating Vanishing Point

- The location of the vanishing point – Obtained using the the camera calibration matrix.

A point at infinity

\[
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
s_j u_1^j \\
s_j u_2^j \\
s_j \\
\end{bmatrix} =
\begin{bmatrix}
q_{11} & q_{12} & q_{13} & q_{14} \\
q_{21} & q_{22} & q_{23} & q_{24} \\
q_{31} & q_{32} & q_{33} & q_{34} \\
q_{41} & q_{42} & q_{43} & q_{44} \\
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
\end{bmatrix} =
\begin{bmatrix}
q_{11} \\
q_{21} \\
q_{31} \\
\end{bmatrix}
\]
Vanishing Lines

The vanishing point concept extends to vanishing lines.

.ml All these vanishing points will be collinear in the image → Forming the vanishing line.

.ml Alternatively, a set of parallel planes in the scene → Their images should all overlap to form one identical line, namely the vanishing line.

.ml Geometrically, the vanishing line – The intersection of the image plane with one of the parallel planes passing through the camera’s optical centre
Vanishing Lines

Vanishing points and vanishing lines are both camera dependent.

**Figure:** Vanishing lines.
The horizon is the vanishing line for the ground plane.

A point below this line will be projected to a point below the horizon while anything above is projected above the horizon.

The horizon is a property of the camera – two viewers at different heights will perceive different horizons.
Figure: Horizon.
Determining Relative Dimensions

**Figure:** Determining relative dimensions – Left: Setup; Right: Geometry and vanishing lines.
Let the point $h_r$ denote the given reference height.

The intersection of the line through the base points $b_r$ and $b_u$ with the vanishing line is vanishing point.

A line from $h_r$ to this vanishing point represents a line that is parallel (in the 3D world) to the line through the two base points $b_r$ and $b_u$.

The point $i$, which is the intersection of this line with the line joining $b_u$ and $h_u$.
- If $i - b_u = h_r - b_r \rightarrow h_u - b_u$ computable from $rac{h_u - b_u}{i - b_u} (h_r - b_r)$. However, this is not the case since the ratios of lengths are not preserved under perspective.

- Geometric invariant – A property of the scene that is invariant (independent of) the perspective projection.
Geometry of Finding a Target Object Size

Figure: The geometry of finding a target object size relative to a reference object.
Projective Invariants

- An **invariant** of a geometric configuration – A function of the configuration whose value is unchanged by a particular transformation.

- ⇒ The distance between two points, or the angle between two line segments, are unchanged under a Euclidean transformation (translation or rotation).
Cross-ratios

Suppose we are given a configuration of four collinear points.

Figure: Ratios of ratios of lengths are invariant.
Cross-ratio of Four Points on a Line.

Many ways of forming the cross ratio between four points.

One simply selects one point, say $X^1$, as a reference point and compute the ratio of distances from that point to two others, say $X^3$, and $X^4$.

Next, compute the ratio of distances from the remaining point $X^2$, to the same two points.

The ratio of these ratios is invariant to perspective transformation as:

$$I = \frac{\delta_{13}}{\delta_{14}} \frac{\delta_{23}}{\delta_{24}} = \frac{\delta_{13}\delta_{24}}{\delta_{14}\delta_{23}}$$

where $\delta_{ij} = \|X^i - X^j\|$.
Figure: The ratios of ratios of lengths are preserved.
Ratios

\[
\frac{|T - B|}{|R - B|} \frac{|\infty - R|}{|\infty - T|} = \frac{H}{R}
\]

scene cross ratio

\[
\frac{|t - b|}{|r - b|} \frac{|v_z - r|}{|v_z - t|} = \frac{H}{R}
\]

image cross ratio
Geometric Invariants

We can take $C$, $h_u$, $i$ and $b_u$ as our four collinear points and form the cross ratio measured in the image

$$I = \frac{(h_u - b_u)(C - i)}{(h_u - i)(C - b_u)}$$

$$ (h_u - i)_{3D} = I \frac{(C - b_u)_{3D}}{(C - i)_{3D}} (h_u - b_u)_{3D}$$

where

- $(h_u - b_u)_{3D}$ - The height of reference object in 3D
- $(C - b_u)_{3D}$ - The height of camera from the reference plane
- $(C - i)_{3D}$ - The difference btw reference height and camera height
Possible to undo the perspective distortion of a plane in the scene if:

- The vanishing line of the plane is known
- Have two reference measurements of known lengths, or angles, in the scene.
Undoing Perspective Distortion

Possible to undo the perspective distortion of a plane in the scene if:

- The vanishing line of the plane is known
- Have two reference measurements of known lengths, or angles, in the scene.
The line passing thru $x$ and $y$ is:

$$x \times y = \text{det} \begin{bmatrix} i & j & k \\ x_1 & x_2 & 1 \\ y_1 & y_2 & 1 \end{bmatrix}$$  \hspace{1cm} (2)$$
Duality between lines and points in homogeneous coordinates:

- The intersection of two lines $l_1$ and $l_2$ is obtained via the cross product $x = l_1 \wedge l_2$
- Generates a vector that is $\perp$ to both $l_1$, $l_2$.
- Hence it must lie in both planes $\rightarrow x \in l_1 \cap l_2$

The line passing thru $x$ and $y$ is:

$$x = \det \begin{bmatrix} i & j & k \\ l_{11} & l_{12} & 1 \\ l_{21} & l_{22} & 1 \end{bmatrix}$$

(3)
Image Plane Projection

Figure: Projecting a plane onto an image.
Stages of Image Plane Projection

- **Similarity transformation** (Rotation, translation and scaling) – Looking directly down on a rotated and translated scene with orthographic projection (infinite focal length). There may be a scale change involved, depending on how far away the viewer is.

- **Affine transformation** (Shearing and aspect ratio distortion) – Looking at the scene obliquely with orthographic projection – hence resulting in foreshortening and shearing (but parallel lines remain parallel). It has 2 DOF – $\alpha$ - the scaling in $x_1$ relative to $x_2$, and $\beta$ - the cotangent of the shearing angle $\theta$.

- **Perspective projection** (Projection with a finite focal length) – Keeping the same oblique viewing angle but with a finite focal length
Affine Transformation

Figure: Projecting a plane onto an image.
Affine Transformation

\[
\begin{bmatrix}
\alpha & \beta & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
1 \\
\end{bmatrix}
= 
\begin{bmatrix}
\alpha x_1 + \beta x_2 \\
x_2 \\
1 \\
\end{bmatrix}
\quad (4)
\]
Perspective Projection

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\mu & \nu & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
1
\end{bmatrix}
= 
\begin{bmatrix}
x_1 \\
x_2 \\
\mu x_1 + \nu x_2 + 1
\end{bmatrix}
= 
\begin{bmatrix}
\frac{x_1}{\mu x_1 + \nu x_2 + 1} \\
\frac{x_2}{\mu x_1 + \nu x_2 + 1} \\
1
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
-\mu \\
-\nu \\
1
\end{bmatrix}
\]

represents the homogeneous coordinates of the vanishing line of the plane.
Inversing Perspective Projection

- In order to inverse this transformation and recover the the view of the plane upto affine transformation $\rightarrow$ Apply inverse perspective projection map.

- The **rectification** (a plan view of) of a surface in an image: Obtained via finding its vanishing line – whose equation then enables the determining the inverse of the perspective transformation map.

- The result will be an image that will still have an affine transform. If one can find two constraint conditions, such as knowledge of the ratio of two lengths in the image, and/or knowledge of an angle between line segments in the image, one can then solve for the two degrees of freedom in the affine transformation.
