EE 653 - Constrained Extrema

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Constrained Extrema

Preliminaries

Constraints

Variables are not independent!

Constrained Optimization

Problem: Let $J : R^n \rightarrow R$

Find $u \in \arg\min J(u)$ s.t. $f(u) = 0$

Approach: Convert to unconstrained optimization

- Elimination - Simple, but not general!
- Lagrange Multipliers $\leftarrow$ IFT

Implicit Function Theorem (IFT)

Theorem: Let $g : R^m \times R^{n-m} \rightarrow R^m$. Then

$$g(x, y) \in R^m \text{ where } x \in R^m,\ y \in R^{n-m}$$

Assume x and y are cont. differentiable. Suppose

$$\exists (x_0, y_0) \text{ s.t. } g(x_0, y_0) = 0$$

and

$$Det D_x g(x_0, y_0) \neq 0$$

Then, $\exists\ h : R^{n-m} \rightarrow R^m$ s.t. $x_0 = h(y_0)$ and $\forall y \in B_\epsilon(y_0)$ $g(h(y), y) = 0$.

Constrained Extrema

Let $\mathcal{W} \subseteq C\left[R, R^{n+m}\right]$ be a function space.

Problem:

$$w^* \in \arg\min_{w \in \mathcal{W}} J(w) = \int_{t_0}^{t_f} g(w, \dot{w}, t)dt$$

subject to a set of n point constraints

$$f_i(w(t), t) = 0 \quad i = 1, \ldots, n$$